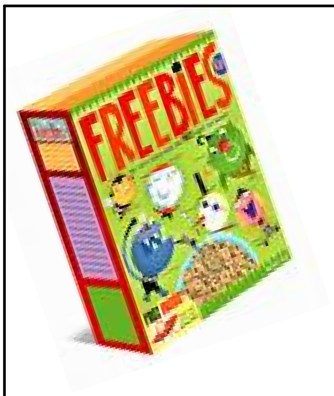
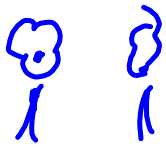


Surface Area, Volume & Capacity

Introduction to Chapter 6... page 222

- Math on the Job... page 224:



match the pattern

A standard roll of antique wallpaper measures 21″ wide and 21′ long, with the 21′ length plastered vertically. Becky needs to completely paper the following walls:

Wall 1: 14 feet wide by 12 feet high

Wall 2: 16 feet wide by 12 feet high

Wall 3: 10 feet wide by 12 feet high

Wall 4: 20 feet wide by 12 feet high

1. How many rolls will Becky need to cover each wall?
2. What is the minimum number of rolls Becky will need to order to cover all of these walls?

As an i... regular... area of...

$$21'' \times \frac{1\text{ ft}}{12''} = 1.75\text{ ft}$$

$$(1.75')(21') = 36.75\text{ ft}^2$$

$$\text{Wall 1} = 14' \times 12' = 168\text{ ft}^2$$

Rolls

$$\frac{168}{36.75} = 4.6$$

SOLUTION

1. To calculate the number of wallpaper rolls needed, first calculate the surface area of one roll of wallpaper.

   Convert the width to feet.
   21 in ÷ 12 in/ft = 1.75 ft

   SA = width × length
   SA = 1.75 × 21
   SA = 36.75 sq. ft.

   Calculate the area of each wall.

   Wall 1:
   SA = width × length
   SA = 14 × 12
   SA = 168 sq. ft.

   Number of rolls to cover Wall 1:
   168 ÷ 36.75 ≈ 4.6

   Wall 2:
   SA = width × length
   SA = 16 × 12
   SA = 192 sq. ft.

   Number of rolls to cover Wall 2:
   192 ÷ 36.75 ≈ 5.2

   Wall 3:
   SA = width × length
   SA = 10 × 12
   SA = 120 sq. ft.

   Number of rolls to cover Wall 3:
   120 ÷ 36.75 ≈ 3.3

   Wall 4:
   SA = width × length
   SA = 20 × 12
   SA = 240 sq. ft.

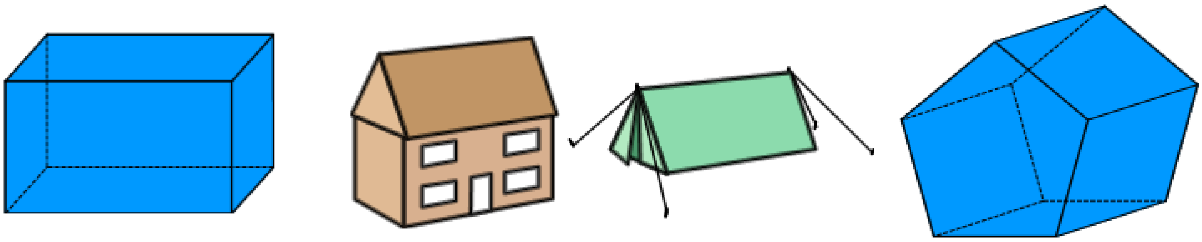
   Number of rolls to cover Wall 4:
   240 ÷ 36.75 ≈ 6.5

2. Total rolls:
   4.6 + 5.2 + 3.3 + 6.5 = 19.6

   Becky will need at least 20 rolls of wallpaper.

3 Dimensional Shapes...

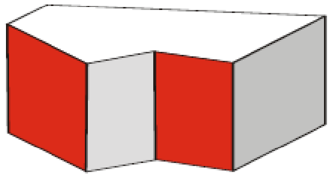
- **Prism -** a 3D shape with ends that are congruent polygons and with sides that are parallelograms.
  ex: rectangular prism; triangular prism



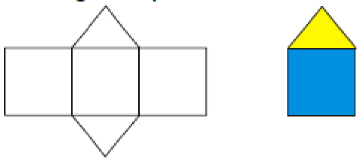
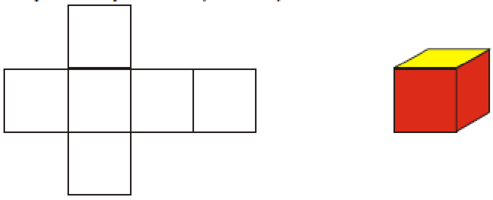
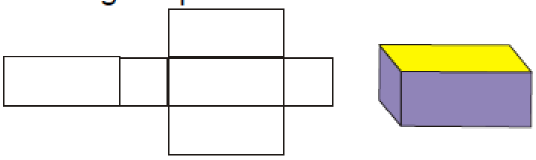
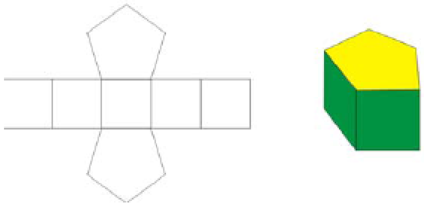
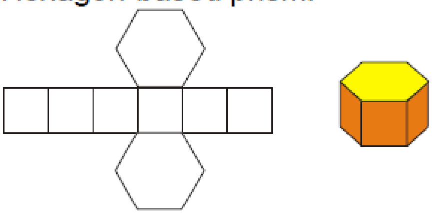
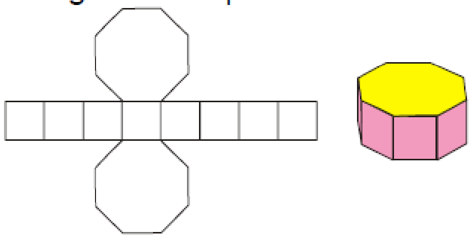
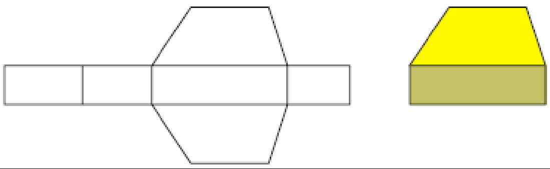
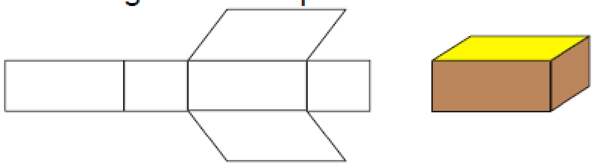
- **Base - o**ne of the parallel faces of a prism
- **Lateral Face -** a face that connects the bases of a prism.

4.12.3: Right Prisms and Their Nets (Teacher)

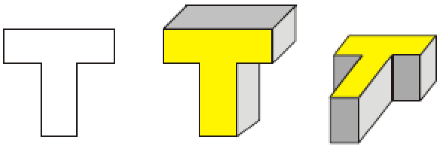
A right prism is a prism with two congruent polygon faces that lie directly above each other.
The base is the face that "stacks" to create the prism. This face determines the name of the prism.



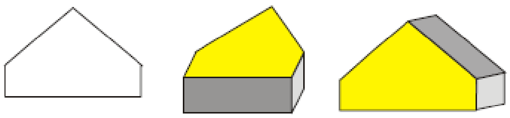
Some right prisms and their nets:

Triangular prism: 	Square prism (cube): 
Rectangular prism: 	Pentagon-based prism: 
Hexagon-based prism: 	Octagon-based prism: 
Trapezoid-based prism: 	Parallelogram-based prism: 

Right prisms with bases that are composite figures:



Composite figure   Right prism



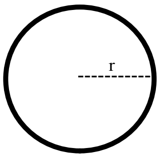
Composite figure   Right prism

REVIEW: Area Formulas...

Rectangle or Square

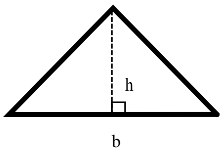
$A = bh$

Circle



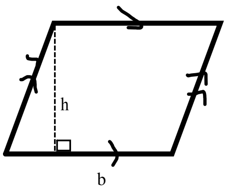
$A = \pi r^2$

Triangle



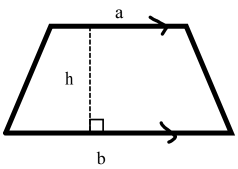
$A = \frac{1}{2} bh$

Parallelogram or Rhombus



$A = bh$

Trapezoid



$A = \frac{1}{2} h(a + b)$

Perimeter and Circumference

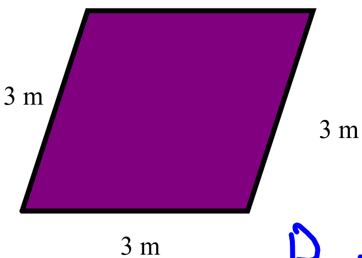
The perimeter is the distance around an object.

Ex: What is the perimeter of the following shapes?

Circumference

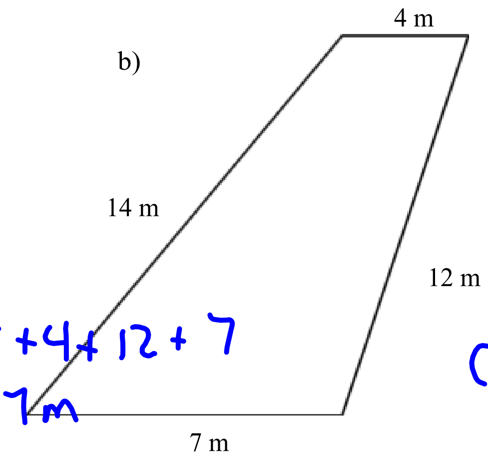
$2\pi r$ OR πd

a)



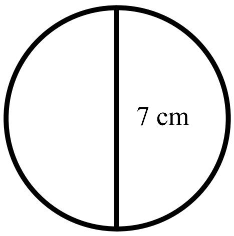
$P = 3(4)$
$= 12m$

b)



$P = 14 + 4 + 12 + 7$
$= 37m$

c)

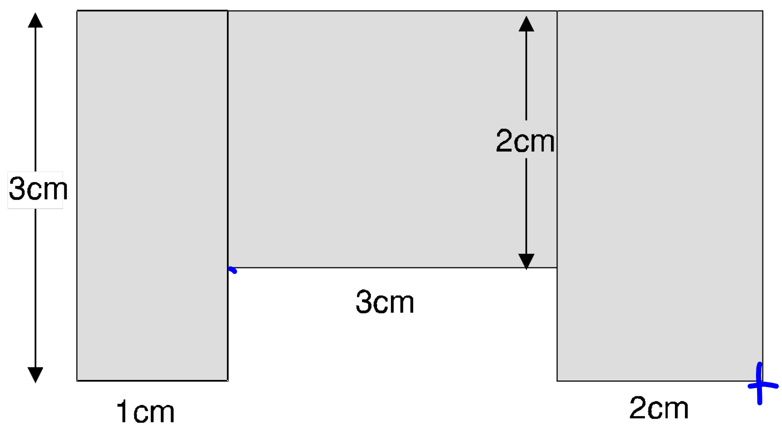


$C = \pi d$
$= 7\pi$

Perimeter and area

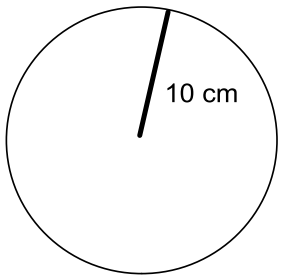
1)Find the perimeter of each figure.
2)Find the area of each figure - they have been divided into rectangles for you.

$P = 3+1+3+2+3$
$+2+1+3+1+1$
$= 20cm$



Find the Perimeter and Area of each shape...

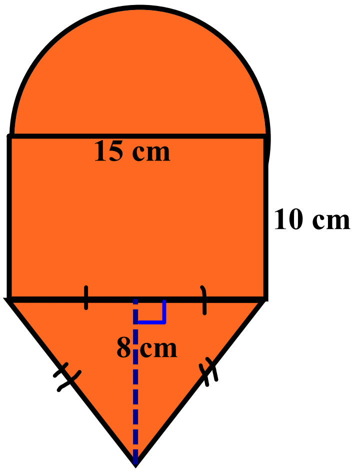
1)

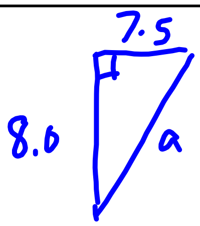


$C = 2\pi r$
$= 2\pi(10)$
$= 20\pi$ cm

$A = \pi r^2$
$= \pi(10)^2$
$= 100\pi$ cm^2

2.

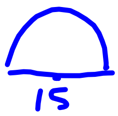




$a^2 = 8^2 + 7.5^2$

$= 120.25$

$a = 10.965$ cm



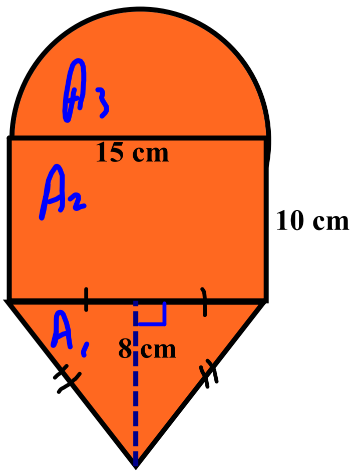
$C = \frac{\pi d}{2}$

$= \frac{\pi(15)}{2}$

$= 23.56$ cm

$P = 10 + 10.965 + 10.965 + 10 + 23.56$

$= 65.49$ cm



$$A_T = A_1 + A_2 + A_3$$

$$= \frac{bh}{2} + lw + \frac{\pi r^2}{2}$$

$$= \frac{(15)(8)}{2} + (10)(15) + \frac{\pi(7.5)^2}{2}$$

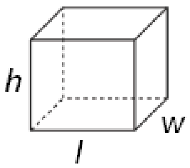
$$= 60 + 150 + 88.36$$

$$= 210 + 88.36$$

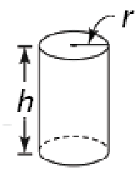
$$= 298.36 \text{ cm}^2$$

Activate Prior Learning:

Surface Areas of Right Prisms and Cylinders



$\boxed{SA = 2wl + 2hl + 2hw}$



$\boxed{SA = 2\pi r^2 + 2\pi rh}$

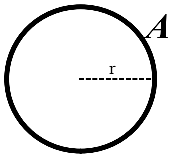
Area Formulas

Rectangle or Square

$\boxed{A = bh}$

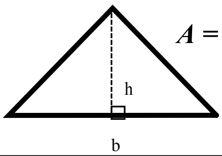
Circle

$A = \pi r^2$

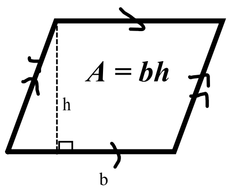


Triangle

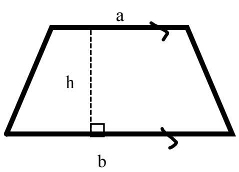
$A = \frac{1}{2} bh$



Parallelogram or Rhombus

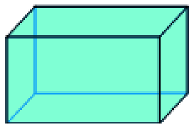


Trapaziod



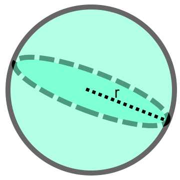
$A = \frac{1}{2} h(a + b)$

Surface Area Formulas



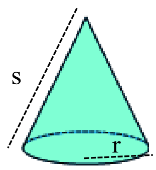
Rectangular Prism

SA = Area of all 6 rectangles added together



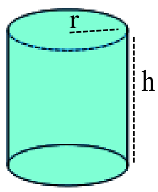
Sphere

$SA = 4\pi r^2$



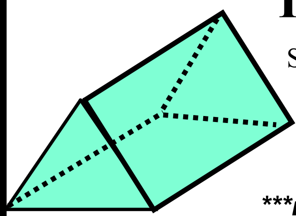
Cone

$SA = \pi r^2 + \pi rs$



Cylinder

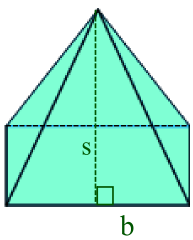
$SA = 2\pi r^2 + 2\pi rh$



Triangular Prism

SA = Area of the three Rectangle Sides
+ Area of two Triangle sides

not all three rectangles have to be the same



Pyramid

SA = Area of the base + all triangular faces

Surface Area

Surface area is the total area of all of the faces of the object.

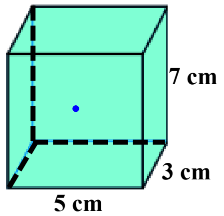
Steps need to find Surface area are...

1. **Draw all of the faces with dimensions displayed on them.**
2. **Find the area of each face.**
3. **Then add up the areas of all of the faces.**

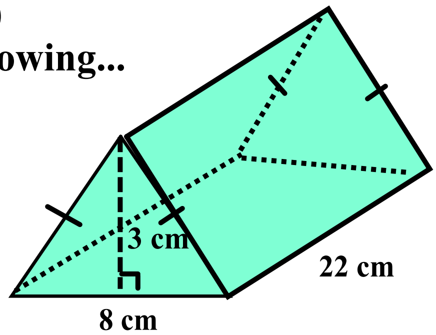
EXAMPLES...

Determine the surface area of each of the following...

1)



2)



$A = 2lw + 2lh + 2wh$

$= (5)(3)(2) + 2(5)(7) + 2(3)(7)$

$= 30 + 70 + 42$

$= 142\,cm^2$

$A_T = 2\ \Delta\text{'s} + 3\text{ rectangles}$

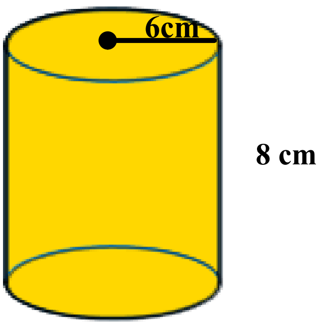
$= \left(\frac{bh}{2}\right)2 + lw + 2lw$

$= \frac{(8)(3)}{2}(2) + (22)8 + 2(5)(22)$

$= 24 + 176 + 220$

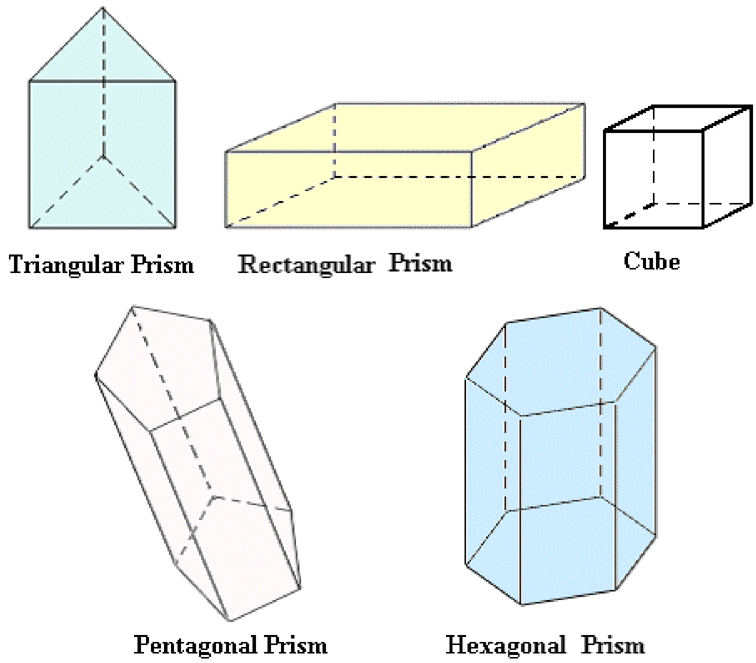
$= 420\,cm^2$

3)



$$A = 2\pi r^2 + 2\pi r h$$
$$= 2\pi(6)^2 + 2\pi(6)(8)$$
$$= 72\pi + 96\pi$$
$$= 168\pi$$
$$= 527.79\ cm^2$$

Regular Polygonal Prism



DISCUSS THE IDEAS Page 226

CALCULATING SURFACE AREA OF A STAINED GLASS LANTERN

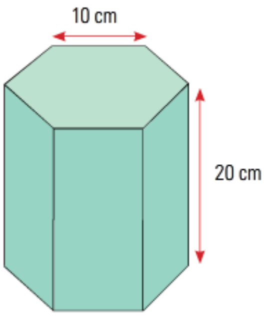
A three-dimensional object is a **prism** if it contains two parallel polygons that are congruent.

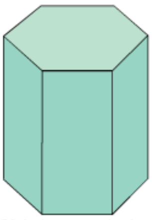
Congruent polygons

- have the same shape and size
- have sides and angles in the same positions

Each of the parallel faces is called a **base**, and the faces connecting the bases are called **lateral faces.**

If the lateral faces are perpendicular to the bases, the prism is a right prism.





Right hexagonal prism

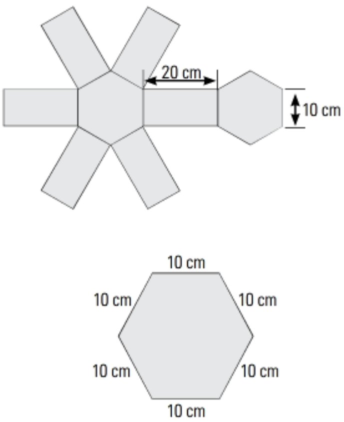
Oblique prism

Shabina makes stained-glass garden lanterns in the shape of hexagonal prisms. The bases (top and bottom) are hexagons and are made of metal. The faces are coloured glass.

1. Look at the lateral faces of the right hexagonal prism above. Draw a net of the right hexagonal prism.
2. If each edge of the hexagonal base is 10 cm long, and the lanterns are 20 cm high, what is the area of each piece of glass used as a lateral face?
3. What is the total surface area of glass that Shabina needs for one lantern? Can you think of two ways to calculate the total surface area?

SOLUTIONS

1. The faces are rectangles.



2. The hexagonal lantern will have six sides. Each side will be rectangular and have a width of 10 cm (the edge length of the hexagonal base) and a length of 20 cm.

   $A = \ell \times w$
   $A = 20 \times 10$
   $A = 200$

   The area of each piece of stained glass will be 200 cm^2.

3. The total area is the sum of areas of all the lateral faces. A hexagonal prism has 6 lateral faces.

   Total area = 6×200
   Total area = 1200 cm^2

DISCUSS THE IDEAS Page 230

SURFACE AREA OF CABINETS

1. Karl is building a set of cabinets. He makes the first one 40 cm long × 40 cm deep × 70 cm high. When the cabinet is fully closed, it is the shape of a rectangular prism. What is the surface area of the cabinet?
2. Karl makes the second cabinet 2 times as long, but with the same depth and height. What is the surface area of the second cabinet?
3. Karl makes the third cabinet 2 times as long and 2 times as high as the first one, but with the same depth. What is the surface area of the third cabinet?
4. Examine the following table showing how the surface area changes when the dimensions are changed.

SURFACE AREA RATIOS

Cabinet	*Length*	*Depth*	*Height*	*Surface area*	$\frac{\text{(Surface Area)}}{\text{(Surface Area of Cabinet 1)}}$
1	L	D	H	14 400	1.00
2	2L	D	H	23 200	1.61
3	2L	D	2H	40 000	2.78
4	2L	2D	2H	?	?

What do you notice about the surface area ratio when the length is doubled? How do the surface areas compare when length and height are doubled?

5. Calculate the surface area of a cabinet where all three dimensions are double those of cabinet one, and calculate the ratio of surface areas. What can you conclude about the relationship between the scale factor used to create the new dimensions and the ratio of the surface areas?

CONCLUSION...

If ALL dimensions are doubled, then the total surface area will be quadrupled.

SOLUTIONS

1. The surface area of a rectangular prism can be expressed by the following equation:

   $SA = 2 \times (\text{length} \times \text{depth}) + 2 \times (\text{length} \times \text{height}) + 2 \times (\text{depth} \times \text{height})$

   The factor of 2 is introduced because the prism has three sets of identical pairs of faces.

   $SA = 2 \times (40 \times 40) + 2 \times (40 \times 70) + 2 \times (40 \times 70)$

   $SA = 14\ 400\ \text{cm}^2$

2. $SA = 2 \times (80 \times 40) + 2 \times (80 \times 70) + 2 \times (40 \times 70)$

   $SA = 23\ 200\ \text{cm}^2$

3. $SA = 2 \times (80 \times 40) + 2 \times (80 \times 140) + 2 \times (40 \times 140)$

   $SA = 40\ 000\ \text{cm}^2$

4. When the length is doubled, the surface area ratio increases by more than half the original ratio. The surface area does not double when one dimension is doubled. When two dimensions are doubled, such as the length and the height, the surface area more than doubles, measuring 40 000 cm^2.

5. $SA = 2 \times (80 \times 80) + 2 \times (80 \times 140) + 2 \times (80 \times 140)$

   $SA = 57\ 600\ \text{cm}^2$

   Ratio of surface areas $= \frac{57\ 600}{14\ 400}$

   Ratio of surface areas = 4.0

   Because the total surface area is a sum of several areas, and not all the terms of the sum are affected when one or two dimensions are doubled, the effect on surface area of changing only one or two dimensions is not an integer. When all three dimensions are doubled, all terms of the surface area sum are affected, and the total surface area scales by the square of the multiplication factor.

HOMEWORK...

Review - Prior Knowledge for Section 6.1.pdf

BLACKLINE MASTER 6.9: SOLUTIONS

Order of Operations

1. $5^2 \times 3 - (84 - 37)$

$= 25 \times 3 - 47$
$= 75 - 47$
$= 28$

2. $(22 - 25)^3 \div [(13 - 7) + 3]$

$= (-3)^3 \div (6 + 3)$
$= -27 \div 9$
$= -3$

3. $\left(\frac{36}{9}\right)^2 \times 2 - 15 \div (-3)$

$= 4^2 \times 2 - 15 \div (-3)$
$= 16 \times 2 - (-5)$
$= 32 + 5$
$= 37$

4. $(-4)^3 + (5 - 11)^2 \div 12 + 20$

$= -64 + (-6)^2 \div 12 + 20$
$= -64 + 36 \div 12 + 20$
$= -64 + 3 + 20$
$= -41$

Finding the Area of Composite Figures

5. $A = \ell w$

$A = (10.5)(4.5)$

$A = 47.25 \text{ in}^2$

6. $A = wh$

$A = (12)(18)$

$A = 216 \text{ cm}^2$

7. $A = \pi r^2$

$A = \pi(3.5)^2$

$A \approx 38.48 \text{ yd}^2$

8. $A = \frac{1}{2}bh$

$A = \frac{1}{2}(5)(2.9)$

$A = 7.25 \text{ ft}^2$

Working with Formulas

9. $4\pi r^2$ $(r = 3.4)$

$= 4\pi(3.4)^2$
≈ 145.27

10. $\frac{1}{3}\pi r^2 h$ $(r = 5.2, h = 8)$

$= \frac{1}{3}\pi(5.2)^2(8)$

≈ 226.53

11. $\pi rs + \pi r^2$ $(r = 3, s = 4.3)$

$= \pi(3)(4.3) + \pi(3)^2$
$\approx 40.53 + 28.27$
≈ 68.8

12. $2\pi r^2 + 2\pi rh$ $(r = 6.7, h = 12.3)$

$= 2\pi(6.7)^2 + 2\pi(6.7)(12.3)$
$\approx 282.05 + 517.80$
≈ 799.85

Converting Measurements Within and Between the SI and Imperial Systems

13. 4.56 km; metres

1 km = 1000 m
4.56 km = 4560 m

14. 56.64 yd; inches (1 yard = 36 inches)

1 yard = 36 inches
56.64 yards = 2039.04 inches

15. 27.2 feet; cm (1 foot ≈ 30.48 cm)

1 foot ≈ 30.48 cm
27.2 feet ≈ 829.056 cm

16. 89.2 miles; km (1 mile = 1.609344 km)

1 mile = 1.609344 km
89.2 miles ≈ 143.55 km

Homework...

6.1 Worksheet - Surface Area of Prisms, Pyramids and Cylinders.pdf

Remember...

SA_{prism} = Add the area of all the faces

$SA_{pyramid}$ = Add the area of a base and the area of the triangular faces (note: $A_{triangle} = bh/2$)

$SA_{cylinder} = 2\pi r^2 + 2\pi rh$

Homework...

p. 232: #1 - 7

6.1 - Build Your Skills Solutions.pdf

Attachments

Section 6.1 Surface Area of Prisms.pdf

Review - Prior Knowledge for Section 6.1.pdf

6.1 Worksheet - Surface Area of Prisms, Pyramids and Cylinders.pdf

6.1 - Build Your Skills Solutions.pdf